A CAD based Computer-Aided Tolerancing Model for the Machining Process

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TR-20040521-1

May 21 2004

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CHAPTER 1

INTRODUCTION

In the design phase of the manufacturing process, products are specified with nominal dimensions and tolerances using the Computer Aided Design (CAD) tools. The tolerance means the allowable variability for certain geometric dimensions or forms. For example, a circle can be specified with the center ($X_0 \pm \Delta x, Y_0 \pm \Delta y$) and the radius $R_0 \pm \Delta r$, where (X_0, Y_0) is the nominal center position, R_0 is the nominal radius, ($\pm \Delta x, \pm \Delta y$) and $\pm \Delta r$ are the tolerances. If the product is manufactured and measured within the tolerance range, then it is deemed a good product. Otherwise, it is a bad product. Therefore, it is very important to understand the relationship between the design and manufacturing in terms of tolerance specification.

Currently, the specifications heavily rely on engineering requirements, experience, and manual calculation. The influence of the manufacturing process on tolerances has not yet been well understood. As a result, the tolerance specifications have to be revised from time to time in the manufacturing phase following the design phase. This type of changes may lead to negative impact on industrial operations. Therefore, it is very meaningful to research and develop scientific methodologies for determining the tolerance specifications during the design phase. The development and application of a CAD based computer aided tolerancing model for the machining process (a typical type of manufacturing process) is chosen as the research topic.

To better explain the research topic, the following sections present the literature survey of the machining process, the geometric dimensions and tolerances, and the computer aided tolerancing techniques for the machining process.

1.1 THE MACHINING PROCESS

Machining (or cutting) refers to a material removal process from a workpiece to form a new surface using various cutting tools [Kalpakjian, 1997]. Commonly seen the machining process include milling and drilling. Milling generates flat, angular, or curved surfaces by cutting away material through the relative movement (namely, feeding) between a workpiece and a rotating multiple-tooth cutting tool. Milling is generally conducted on a milling machine which holds the workpiece, rotates the cutter, and feeds the workpiece. Figure 1-1 shows a milling machine and some milling operations. A drilling process, on the other hand, forms a hole in the workpiece. Drilling can be performed on different machines, including the milling machine.
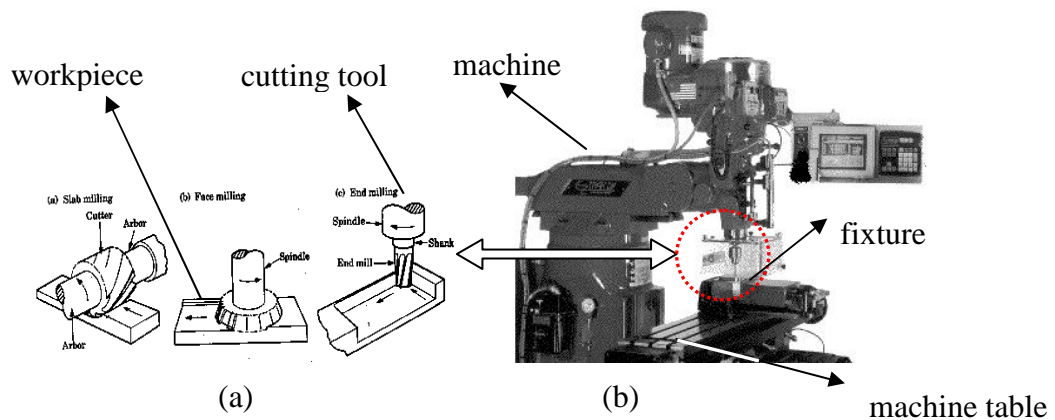
Fig. 1-1 Milling operations and a milling machine

During the machining operation, the workpiece is located and fixed on a holding device, called a fixture. The fixture is located on the machine table, which can move

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(together with the workpiece) relative to the rotating cutting tool. The movement of the table and the rotation of the cutting tool are performed by motors on the machine. The speed, the depth and the direction of the movement and rotation can be controlled through manual operations or automation (e.g., computer numerical control or CNC).

Due to the manufacturing imperfection, any of the aforementioned components, such as the workpiece, the fixture, the machine and the cutting tool, can have geometric errors. These geometric errors influence the relative position between the cutting tool and the workpiece and therefore affects the geometric dimensions of the newly generated surface. The influence can be illustrated in Fig. 1-2.

(a) error 1   (b) error 2   (c) ideal
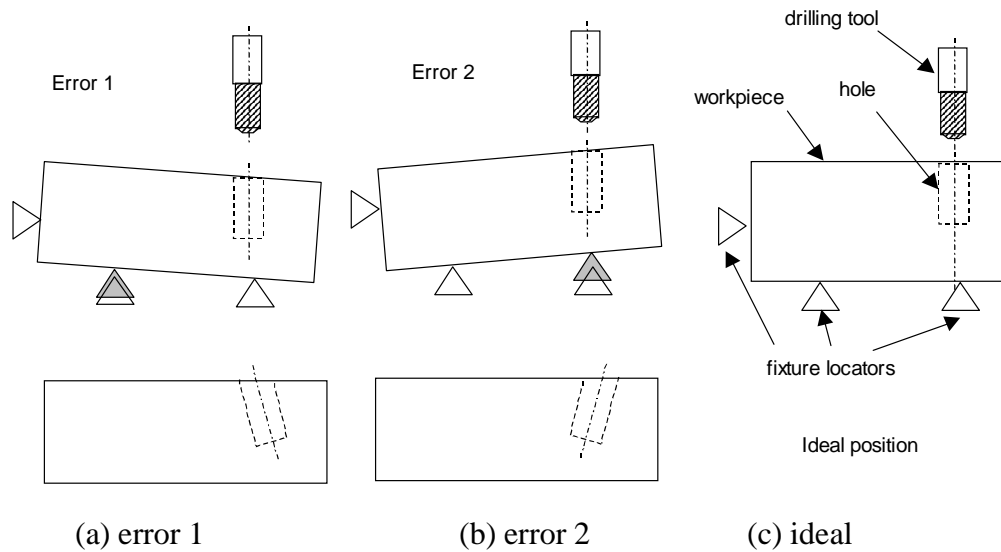
Fig. 1-2 A drilling operation and potential fixture error

Figure 1-2 shows a drilling tool which drills a hole in a block-shaped workpiece. The workpiece is located on the fixture locators. Ideally the hole is designed as in (c), where the hole is perpendicular to the top surface. Due to the fixture locator error, the locators deviate from their ideal position (e.g., (a) and (b)). As a result, the holes

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generated in (a) and (b) situations will not be perpendicular to the top surface. This means that errors from the fixture are transferred into the newly generated holes.

1.2 GEOMETRIC DIMENSIONS AND TOLERANCING

The geometric dimension represents the ideal, or required size while the tolerances represent the allowable range of a certain dimension. The commonly used tolerances are defined by the ANSI Y14.5M 1982 standard. These tolerances and their definitions can be found in [Chang et al., 1998] and some examples are illustrated in Table 1-1. Further explanation can be found in Chapter 3.

Table 1-1 Examples of tolerance definition

Tolerance	Symbol	Definitions
Straightness	——	A condition where an element of a surface or an axis is a straight line.
Flatness	▱	A two dimensional tolerance zone defined by two parallel planes within which the entire surface must lie.
Parallelism	//	The condition of a surface or axis which is equidistant to all points from a datum of reference.
Angularity	∠	The distance between two parallel planes, inclined at a specified basic angle in which the surface, axis, or center plane of the feature must lie.
True Position	⊕	A zone within which the center, axis, or center plane of a feature of size is permitted to vary from its true (theoretically exact) position.
Roundness	○	A circularity tolerance specifies a tolerance zone bounded by two concentric circles within which each circular element of the surface must lie, and applies independently at any plane.

1.3 COMPUTER AIDED TOLERANCING

Computer Aided Tolerancing (CAT) has been a recent research focus to solve the tolerancing problems with the help of the computer modeling and analysis. The research directions in this field include the tolerance analysis and the tolerance synthesis [Dong, 1997]. In general, the tolerance analysis studies the tolerance stack-up (or the final tolerance) given the individual component tolerances. The tolerance synthesis optimizes the individual component tolerances given the specification of the final tolerance. This study focuses on the tolerance analysis.

Commonly used methods for tolerance analysis include the Worst Case Method (WCM) and statistical methods such as the Root Sum Square (RSS) method and the Monte Carlo simulation (MCS) [Zhong, 2002]. The WCM adds up (i.e., stacks up) individual tolerances assuming each component is specified with its extreme dimensions. The statistical methods assume a distribution of each individual dimension. This assumed distribution can be a normal distribution or others. In general, the statistical methods can generate a more reasonable tolerance than WCM since the latter model assumes extreme conditions that rarely happen. Statistical methods will be applied in this study.

CAT is applicable to various manufacturing processes such as the machining process, which in general refers to a material removal process from a workpiece using cutting tools. The process usually involves a workpiece, a cutting tool (such as a milling tool or a drilling tool), a fixture (to locate and fix the workpiece), and a machine (to rotate and move the cutting tools and to cut off material from the workpiece.) After the material removal, the workpiece will have a newly formed surface. The geometric

quality of the new surface is often measured by tolerance specifications, such as flatness and parallelism, and so on.

Geometric dimensions and tolerances are specified with more than ten different parameters, such as flatness, parallelism, angularity and so on. However, the published methods for tolerance analysis only deal with a limited number of them. For example, only two types of tolerances, i.e., flatness and parallelism, were modeled in [Zhong, 2002]. The methods in [Huang, 2003; Roy, 1997; Zhou et al., 2003] cannot process the surface geometrics such as flatness. Therefore, there is a need for developing more effective models for tolerance analysis applied to the machining process using CAD tools. Based on the literature survey, the focus of this research is to develop a CAD based CAT model that deals with more applicable tolerance specifications, assuming the machining process variation is known. The process variation includes the variation of the workpiece (before cutting), of the cutting tool, of the fixture, and of the machine. More specifically, this research intends to study the following tolerances: position tolerance, flatness, parallelism, angularity, perpendicularity and roundness. For the purpose of comparison, the following table (Table 1-2) shows the difference between this study and other published papers.

Table 1-2 Comparison between this study and other researches

Related Research	Zhong, 2002	Huang et al., 2003; Zhou et al., 2003	This study
Tolerance Analysis	Flatness, Parallelism	Parallelism, Distance (position)	Position, Flatness, Parallelism, Angularity, Perpendicularity, Roundness

The proposed methodology integrates the CAD tools and methods (e.g., transformation) with the CAT methods (e.g., Monte Carlo simulation) to calculate the final tolerance of a machining process. Compared to those published methods, this study will present a more comprehensive model dealing with more tolerance specifications. The methods presented in this study can be used in the design phase for machining.

1.4 THESIS ORGANIZATION

The remainder of this thesis is organized as follows:

Chapter 2: A tolerance stack-up model is developed in this section to calculate the geometric dimension and tolerances for workpieces. The stack-up model integrates errors from the machine, the tool, the workpiece, and the fixture. There are three steps of developing such a model: (1) representing the workpiece and the processes, (2) calculating the geometric relationship between components using the Homogeneous Transformation Matrix method, and (3) modeling the machining process and generating a new surface on the workpiece.

Chapter 3: A set of algorithms are developed to calculate the tolerances of the machined new surfaces by means of geometric analysis and of Monte Carlo simulation. These tolerances include the Positional tolerance, Flatness, Parallelism, Angularity, Perpendicularity, and Roundness.

Chapter 4: A numerical case study is developed in this chapter to demonstrate the proposed models and algorithms.

Chapter 5: Conclusion remarks are given in this chapter.

CHAPTER 2

TOLERANCE STACK-UP FOR THE MACHINING

PROCESS

This chapter focuses on analyzing the geometric relationship among various components (e.g., the workpiece, the fixture, the machine and the tool) involved in the machining process and developing a tolerance stack-up model to quantify the influence of component errors on the machined workpiece. In general, there are three steps in developing such a model: (1) representing the physical components, e.g., the workpiece and the machine, (2) calculating the geometric relationship between components, and (3) modeling the machining process and generating a new surface on the workpiece.

2.1   REPRESENTING COMPONENTS

Numerous methods, such as 2-D drawings, parametric representation, points representation, and 3-D solid model representation, have been developed to represent the physical components [Chang et al., 1998]. The 2-D representation actually projects a component (physically 3-D) into 2-D spaces. The 2-D representation is very convenient for most engineering drawings. However, the visualization of 2-D drawing is usually less attractive than a 3-D representation.

In the 3-D representation, an object (or component) can be represented by a set of surface points and their coordinates. For example, a block-shaped object can be

represented by a group of coordinates, which can be stored in a matrix. This matrix can

be easily generated from a commercial software.

$$\begin{bmatrix} x_1 & \cdots & x_n \\ y_1 & \cdots & y_n \\ z_1 & \cdots & z_n \end{bmatrix}$$
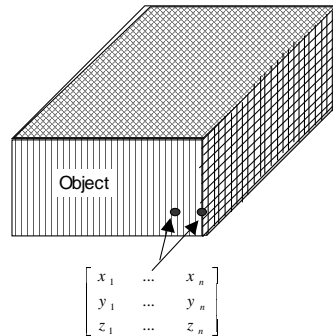
Fig. 2-1 Representation of a 3-D object

2.2   CALCULATING GEOMETRIC RELATIONSHIP

The geometric relationship among different components directly affects the

machining tolerance.   Fig. 2-2 shows an example of a drilling process with two different

fixture locations. In this figure, the right hand side locator has a coordinate set (x_1, δ_1)

while in Fig. 2-2 (b), the corresponding locator has the coordinates (x_2, δ_1), where $x_2 > x_1$.

All other parameters are identical for the two machines. Obviously, the machined holes

will be different in terms of the relative positions between the hole and the top surface,

i.e., $\theta_2 > \theta_1$.   In other words, the resultant error (θ_2, θ_1) is affected by the relative

geometric relations.
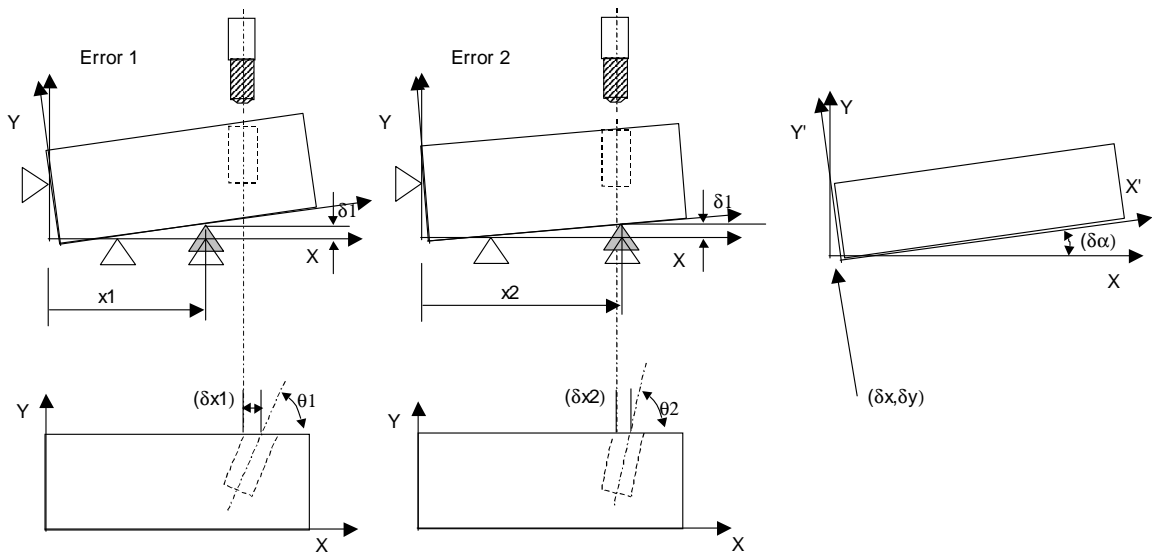
(a) position 1   (b) position 2   (c) rotation and translation
Fig. 2-2 Fixture error and its impact on machining surface

The geometric relationship can be represented by the rotation and translation between these components using their local coordinate systems. For example, the local coordinate system for the workpiece (X-Y) is deviated from its ideal position due to the fixture error. The deviation can be represented as the rotation ($\delta\alpha$) from the axis and the translation ($\delta x, \delta y$) from the origin. Given the rotation and translation, it is possible to calculate the resultant errors such as $\delta x, \delta y$ and $\delta\theta$.

The calculation of resultant errors is based on the Homogeneous Transformation Matrix method [Slocum, 1992], which is essentially the common representation of transformations in the field of computer graphics [Vrajitoru, 2002].

The Homogeneous Transformation Matrix (HTM) is a 4×4 matrix describing the spatial relationship between two objects, which can be vectors or rigid bodies. The "two objects" in our case are the ideal and the actual (with errors) positions of the components.

The HTM represents the linear or angular distance of translation or rotation from one object to the other.

In this study, an individual object is represented as a set of points in the form of a matrix, $\mathbf{X_o}$:

$$X_0 = \begin{bmatrix} x_1 & \cdots & x_n \\ y_1 & \cdots & y_n \\ z_1 & \cdots & z_n \\ 1 & 1 & 1 \end{bmatrix}_0 \qquad (2\text{-}1)$$

where the $\begin{bmatrix} x_1 & y_1 & z_1 & 1 \end{bmatrix}_0'$ represents the homogeneous coordinates of a certain point on the object. Let $\mathbf{X_o}$ represent the ideal object and $\mathbf{X_1}$ the object with deviation, then the equation of the transformed points according to the HTM can be expressed as follows:

(1) rotation around the X-direction (R_x)
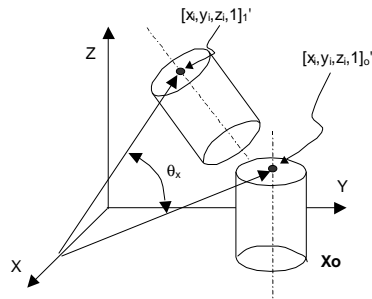
Fig. 2-3 Rotation around the X-direction (R_x)

$$X_1 = R_x X_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta x) & -\sin(\theta x) & 0 \\ 0 & \sin(\theta x) & \cos(\theta x) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} X_0 \qquad (2\text{-}2)$$

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(2) rotation around the Y-direction (R_y)
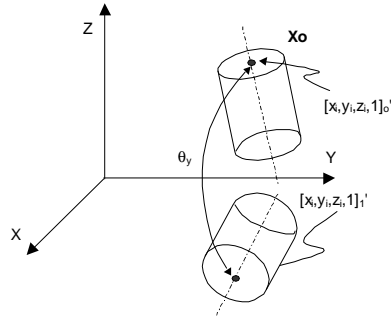
Fig. 2-4 Rotation around the Y-direction (R_y)

$$X_1 = R_y X_0 = \begin{bmatrix} \cos(\theta y) & 0 & -\sin(\theta y) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(\theta y) & 0 & \cos(\theta y) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} X_0 \qquad (2\text{-}3)$$

(3) rotation around the Z-direction (R_z)
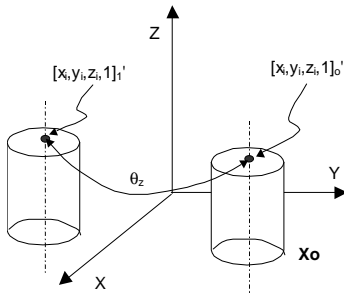
Fig. 2-5 Rotation around the Z-direction (R_z)

$$X_1 = R_z X_0 = \begin{bmatrix} \cos(\theta z) & -\sin(\theta z) & 0 & 0 \\ \sin(\theta z) & \cos(\theta z) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} X_0 \qquad (2\text{-}4)$$
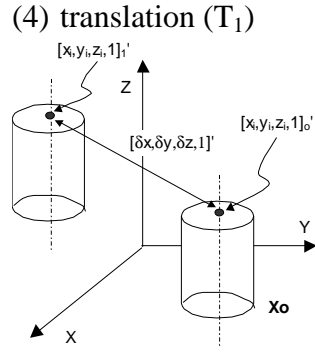
(4) translation (T_1)

Fig. 2-6 Translation of vector ($\delta x, \delta y, \delta z$)

$$X_1 = T_1 X_0 = \begin{bmatrix} 1 & 0 & 0 & \delta x \\ 0 & 1 & 0 & \delta y \\ 0 & 0 & 1 & \delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} X_0 \qquad (2\text{-}5)$$

A general transformation from $\mathbf{X_0}$ to $\mathbf{X_1}$ can be described as a combination of rotations in X-,Y-, and Z-directions and a translation:

$$X_1 = T_1 * R_x * R_y * R_z * X_0 \qquad (2\text{-}6)$$

assuming that the transformation sequence is known. Generally, the above equation is not commutative except that in our case we can assume that we deal with a *small error*. [Greenwood, 1988]. Under the small error assumption, we can use the following approximations,

$$\sin(\delta\theta) \approx \delta\theta$$
$$\cos(\delta\theta) \approx 1 \qquad (2\text{-}7)$$
$$\delta v_1 * \delta v_2 \approx 0$$

Where v_1 and v_2 are any of the δ quantities accruing in equation(2-2) to (2-5).

This study is based on the *small error* assumption and therefore, the Eqn. (2-6) is commutative.

The elements (i.e., $\delta x, \delta y, \delta z, \delta \theta_x, \delta \theta_y, \delta \theta_z$) in the HTMs decide the deviation of a component from its ideal position. These elements are not directly available but can be calculated from the errors of each machining component, e.g., δ_1 in Fig. 2-2. According to [Zhong, 2002], the calculation can be illustrated using a 3-2-1 fixturing scheme.

(a) A 3-2-1 fixturing   (b) The fixture coordinate system
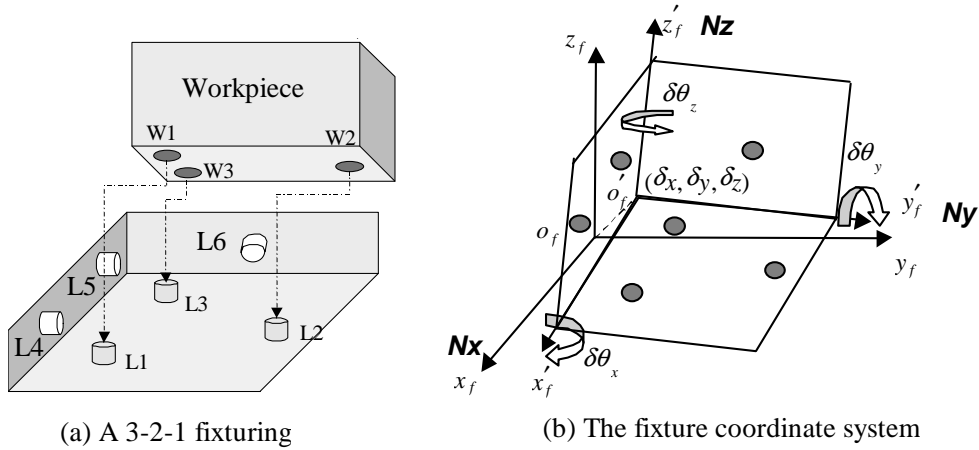
Fig. 2-7 A 3-2-1 fixturing scheme and the related errors

As shown in Fig. 2-7, a 3-2-1 fixturing scheme uses six locators (i.e., $L_1, L_2, L_3, L_4, L_5,$ and L_6) to locate and hold the workpiece. The errors on each locator can be generated using the Monte Carlo simulation given the tolerance of the fixture. The detailed procedures are summarized as follows:

(1) randomly generate the coordinates (e.g., $\vec{L}_i = (x_i, y_i, z_i)'$) of the locators based on the fixture tolerance (i.e., T_f) and the ideal positions (e.g., $\vec{L}_{i,0} = (x_{i,0}, y_{i,0}, z_{i,0})'$), under the assumption of normal distribution. Mathematically,

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$$x_i \sim N(x_{i,0},(T_f/3)^2)$$
$$y_i \sim N(y_{i,0},(T_f/3)^2) \qquad i=1:6 \qquad (\,2\text{-}8)$$
$$z_i \sim N(z_{i,0},(T_f/3)^2)$$

The form of the above equation can be represented as $x \sim N(\mu,\sigma^2)$, which means x is a random variable and follows a normal distribution with a mean of μ and a standard deviation of σ. For a normal distribution, $\mu \pm 3\sigma$ covers 99.73% of data points so that $\pm 3\sigma$ is often used as the tolerance specification (i.e, $\pm T_f$). In this case, $x_{i,0}$ is the designed value and represents the mean for the ith locator. Therefore, $\sigma = T_f/3$ and the representation of the x_i variable becomes $x_i \sim N(x_{i,0},(T_f/3)^2)$.

    (2) calculate the axis directions of the fixture coordinate system (CS_F) based on the generated coordinates. That is,

$$\vec{n}_z = (\vec{L}_1 - \vec{L}_2) \times (\vec{L}_1 - \vec{L}_3)\,/\,\left\|(\vec{L}_1 - \vec{L}_2) \times (\vec{L}_1 - \vec{L}_3)\right\|$$
$$\vec{n}_x = (\vec{L}_5 - \vec{L}_4) \times \vec{n}_z\,/\,\left\|(\vec{L}_5 - \vec{L}_4) \times \vec{n}_z\right\| \qquad (\,2\text{-}9)$$
$$\vec{n}_y = \vec{n}_z \times \vec{n}_x$$

    (3) calculate the deviation of CS_F from its nominal (ideal) position, in the form of ($\delta x, \delta y, \delta z, \delta\theta_x, \delta\theta_y, \delta\theta_z$).

$$\delta\theta_z = \begin{cases} \mathbf{atan}\left(\dfrac{\vec{n}_x(y)}{\vec{n}_x(x)}\right) & \text{when } \vec{n}_x(x) \neq 0 \\[4mm] 0 & \text{when } \vec{n}_x(x) = 0 \end{cases} \qquad (\,2\text{-}10)$$

$$\delta\theta_y = \mathbf{asin}\left(\vec{n}_x(z)\right)$$

$$\delta\theta_x = -\mathbf{asin}\left(\vec{n}_y(z)\right)$$

$$\begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} = -\begin{bmatrix} \vec{n}_z(x) & \vec{n}_z(y) & \vec{n}_z(z) \\ \vec{n}_x(x) & \vec{n}_x(y) & \vec{n}_x(z) \\ \vec{n}_y(x) & \vec{n}_y(y) & \vec{n}_y(z) \end{bmatrix}^{-1} \begin{bmatrix} \vec{n}_z(x)*x_2 + \vec{n}_z(y)*y_2 + \vec{n}_z(z)*z_2 \\ \vec{n}_x(x)*x_4 + \vec{n}_x(y)*y_4 + \vec{n}_x(z)*z_4 \\ \vec{n}_y(x)*x_6 + \vec{n}_y(y)*y_6 + \vec{n}_y(z)*z_6 \end{bmatrix}$$

These variables (i.e., $\delta x, \delta y, \delta z, \delta\theta_x, \delta\theta_y, \delta\theta_z$) can be replaced in Eqn. (2-2)-(2-6) with the value obtained from Eqn. (2-10) to calculate the workpiece position in a coordinate system with errors.

2.3   MODELING THE MACHINING PROCESS

The machining process generates new surfaces on the workpiece which is located on the fixture. As shown in Fig. 2-8, the machining process can be viewed as an operation replacing the workpiece surface (partially) with the tool path.
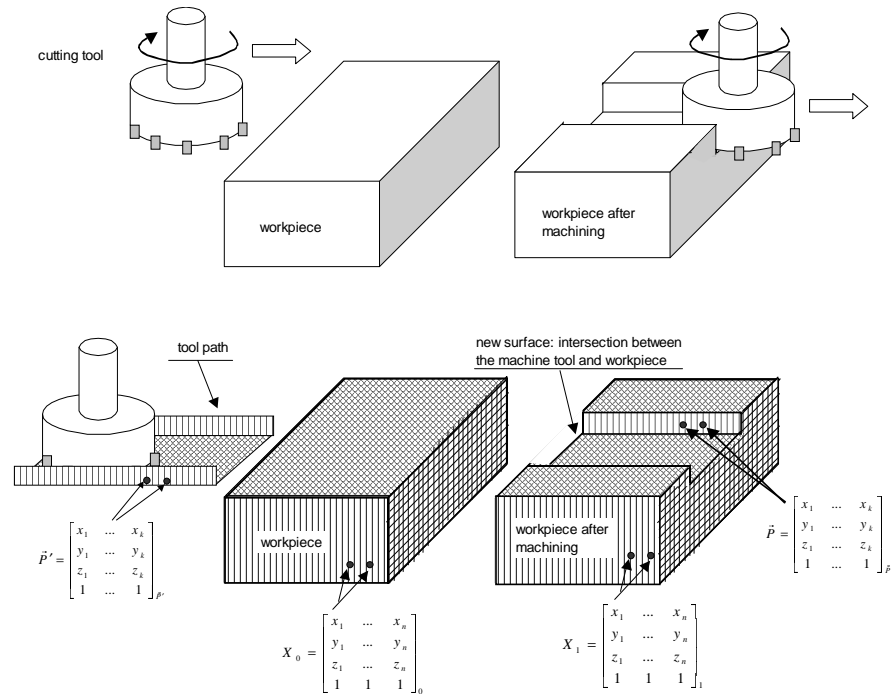
Fig. 2-8 Representation of machining operations

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The replacement can be mathematically modeled using the matrix representation of the workpiece and of the tool path. That is,

$$\vec{P} = I\vec{P}'$$
(2-11)

where I is a 4×4 identity matrix, $\vec{P}$ and $\vec{P}'$ represent the new surface and the ideal tool path. However, due to the errors in each machining component, the representation has to be modified in the following form:

$$\vec{P} = T^{-1}\vec{P}'$$
(2-12)

where T represents the combination of the translation and rotation of the workpiece in machining. Graphically, the relationship can be represented as shown in Fig. 2-9:
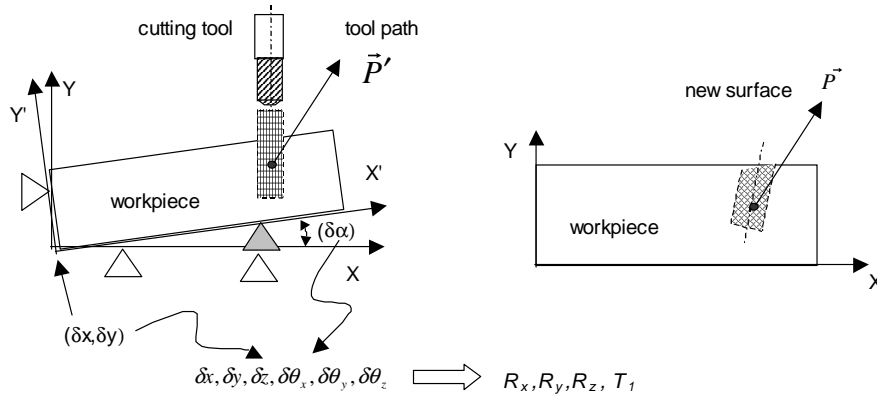
Fig. 2-9 Representation of machining operations

The newly generated surface, represented by $\vec{P}$ can then be used to update the original workpiece matrix $\vec{X}_0$ and form a new matrix $\vec{X}_1$.

CHAPTER 3

ALGORITHMS FOR TOLERANCE ESTIMATION

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In the previous chapter, the workpiece is represented as a set of coordinates of a given number of points on its surface. These coordinates do not represent the specified tolerances for the workpiece. Therefore, it is necessary to convert the coordinates into tolerance specifications to compare them with the designed tolerance based on the ANSI standard. In reality, the process takes two steps: measuring the coordinates of the workpiece and then calculating the related tolerance parameters. As it has been pointed out in the introduction, there are limitations to the existing literature. A detailed discussion of these limitations can be found in Chapter 1. Therefore, a more comprehensive model is needed to calculate the tolerance in the study of computer aided tolerancing. In this chapter, a set of algorithms are developed to calculate the tolerances of the machined new surfaces by means of geometric analysis and of Monte Carlo simulation. These tolerances include the Position, Flatness, Parallelism, Angularity and Perpendicularity.

In this chapter, the computations will be performed based on a particular example of a machined workpiece illustrated in Fig. 3-1.

3.1   PARAMETER ANALYSIS FOR INDIVIDUAL FEATURE

Let us assume a workpiece is represented by a set of coordinates, $\vec{X}$, which is a $4 \times n$ matrix using homogeneous coordinates as shown in Eqn. (3-1). The workpiece has a number of features, including the hole, the cylinder, and the flat surface.
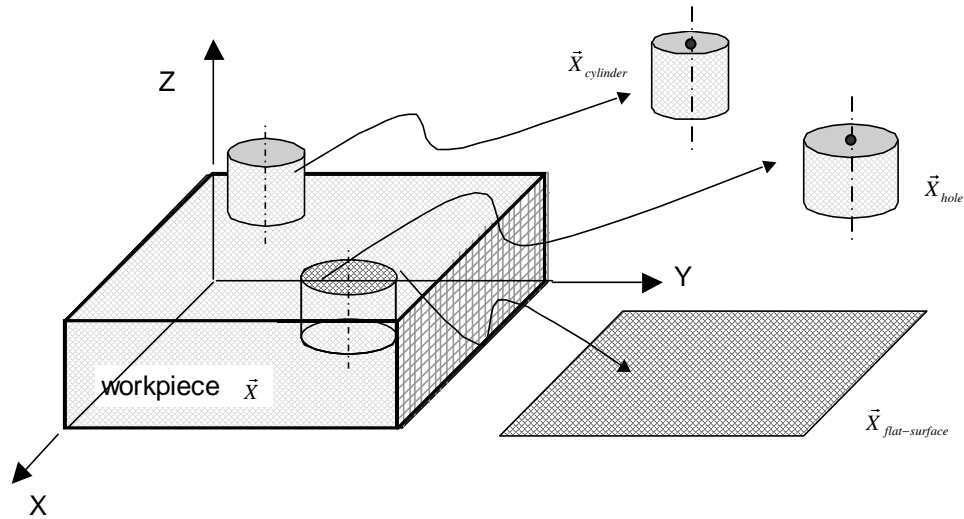
Fig. 3-1 Representation of the workpiece

For convenience, the matrix of the workpiece coordinates, $\vec{X}$, can be partitioned into individual matrices corresponding to each of the feature of the workpiece. $\vec{X}$ is then a concatenation of these individual matrices.

$$
\vec{X} = \begin{bmatrix} x_1 & ... & x_n \\ y_1 & ... & y_n \\ z_1 & ... & z_n \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \vec{X}_{hole} & \vec{X}_{cylinder} & \vec{X}_{flat-surface} & ... \end{bmatrix}
$$

$$
\vec{X}_{featrue} = \begin{bmatrix} x_1 & ... & x_m \\ y_1 & ... & y_m \\ z_1 & ... & z_m \\ 1 & 1 & 1 \end{bmatrix}_{feature} ; feature = hole, cylinder, flat - surface, ...
$$

(3-1)

The calculation of the matrix corresponding to different features requires different equations. In this study, three types of features are discussed: (1) straight line, (2) flat surface, and (3) cylinder/hole. These features can be represented by a set of parameters. As shown in Table 3-1, both the hole feature and the cylinder feature can be represented by the slope of the axis, a point on the axis and the diameter (or radius). For example, a

straight line can be represented as the direction vector (*a, b, c*) of the line and a point

(x_0, y_0, z_0) on this line. The representation can be summarized as follows:

Table 3-1 Parametric representation of features

Feature	Required parameters			Equation for any point (x, y, z) on the feature
Straight Line	The direction of the line, (*a, b, c*)	A point on the line, (x_0, y_0, z_0)		$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + t \begin{bmatrix} a \\ b \\ c \end{bmatrix} ; \ \mathbf{t} \in \mathbf{R} ;$$
Flat Surface	The vector normal to the surface, (*a, b, c*)	A point on the surface, (x_0, y_0, z_0)		$a(x - x_0) + b(y - y_0)$ $+ c(z - z_0) = 0$ or $z = c' + a'x + b'y$, where $a' = -a/c$ $b' = -b/c$ $c' = (ax_0 + by_0 + cz_0)/c$
Cylinder (hole)	The vector of the axis, (*a, b, c*)	A point on the axis, (x_0, y_0, z_0)	The radius of the cylinder, R	$A^2 + B^2 + C^2 = R^2$ $A = c(y - y_0) - b(z - z_0)$ $B = a(z - z_0) - c(x - x_0)$ $C = b(x - x_0) - a(y - y_0)$

Based on the feature coordinates, it is possible to calculate the above parameters

using the linear least square estimation method (LLSE) [Rice, 1988]. These parameters

can then be used to calculate the tolerance.

To explain the LLSE, it is convenient to use a simple example with two variables,

x and y. The observations of x and y are (x_1, y_1), (x_2, y_2), …, (x_i, y_i), …, (x_n, y_n). The

LLSE method is to find a fitted line with a slope *a* and an interception *b* such that the sum

of the distances of each point to the fitted line is a minimum. The relationship between x

and y is represented as:

$$y = a * x + b \tag{3-2}$$
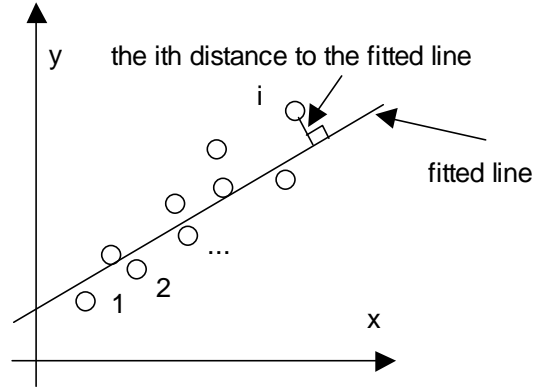
Fig. 3-2 An example of LLSE

In essence, the LLSE is a method to estimate the parameters of a linear model (with one or more x variables), represented as:

$$y = \beta_0 + \beta_1 x_1 + ... + \beta_{p-1} x_{p-1} \tag{3-3}$$

given a set of observations of y_i and of x_i's. In general, the parameters and observations can be represented as the following matrixes:

$$\vec{Y} = \begin{bmatrix} y_1 \\ y_2 \\ ... \\ y_n \end{bmatrix}; \vec{X} = \begin{bmatrix} 1 & x_{1,1} & x_{1,2} & ... & x_{1,p-1} \\ 1 & x_{2,1} & x_{2,2} & ... & x_{2,p-1} \\ ... & ... & ... & ... & ... \\ 1 & x_{n,1} & x_{n,2} & ... & x_{n,p-1} \end{bmatrix}; \vec{\beta}' = \begin{bmatrix} \beta_0 \\ \beta_1 \\ ... \\ \beta_{p-1} \end{bmatrix} \tag{3-4}$$

The LLSE is used to estimate the parameters $\vec{\beta}'$ such that the norm of the difference between the observation $\vec{Y}$ and the prediction $\hat{\vec{Y}}$ calculated with Eqn. (4-2) is a minimum. The solution can be proven as:

$$\hat{\vec{\beta}} = (\vec{X}' \vec{X})^{-1} \vec{X}' \vec{Y} \tag{3-5}$$

In the previous section, the coordinate points are available after simulating the machining process. Therefore, the LLSE method can be implemented to calculate the associated parameters for the line, the plane and the cylinder. For example, the best fit plane can be estimated using the following procedures:

(1) formulating the equation of the plane

$$z = c' + a'x + b'y \tag{3-6}$$

(2) formulating $\vec{X}$

$$\vec{X} = \begin{bmatrix} 1 & x_1 & y_1 \\ ... & ... & ... \\ 1 & x_n & y_n \end{bmatrix} \tag{3-7}$$

(3) formulating $\vec{Y}$

$$\vec{Y} = \begin{bmatrix} z_1 \\ ... \\ z_n \end{bmatrix} \tag{3-8}$$

(4) calculating the parameters in Eqn. (3-5) by solving the following equations

$$\hat{\vec{\beta}} = \begin{bmatrix} \hat{c}' \\ \hat{a}' \\ \hat{b}' \end{bmatrix} = \left(\vec{X}' \vec{X} \right)^{-1} \vec{X}' \vec{Y} \tag{3-9}$$

(5) computing the parameters of the vector of the line (as in Table 3-1)

$$\hat{a} = -\hat{a}' / \left\| (-\hat{a}', -\hat{b}', 1) \right\|$$
$$\hat{b} = -\hat{b}' / \left\| (-\hat{a}', -\hat{b}', 1) \right\| \tag{3-10}$$
$$\hat{c} = 1 / \left\| (-\hat{a}', -\hat{b}', 1) \right\|$$

(6) solving the parameters of a point on the line

$$\hat{x}_0 = \sum_{i=1}^{n} x_i \bigg/ n$$
$$\hat{y}_0 = \sum_{i=1}^{n} y_i \bigg/ n \tag{3-11}$$
$$\hat{z}_0 = \sum_{i=1}^{n} z_i \bigg/ n$$

Following similar procedures to [Muralikrishnan, 2004], the equations to find the parameters for the line and the cylinder can be solved. Based on the calculated parameters, the calculation of the tolerance becomes possible.

3.1.1 Position

The position tolerance refers to a range within which the positional parameters, such as the point, the axis, or the center plane of a feature, are permitted to vary from the designed position. Assuming that the point is specified as ($x_0 \pm \Delta x, y_0 \pm \Delta y, z_0 \pm \Delta z$) and the actual point for a workpiece is estimated as ($\hat{x}_0, \hat{y}_0, \hat{z}_0$), then the piece position fulfills the tolerance requires if:

$$
\begin{aligned}
abs(x_0 - \hat{x}_0) < \Delta x \\
abs(y_0 - \hat{y}_0) < \Delta y \\
abs(z_0 - \hat{z}_0) < \Delta z
\end{aligned}
\tag{3-12}
$$

3.1.2 Flatness

The flatness refers to the minimum distance between two parallel planes enclosing a set of measurement points taken on the surface that is checked for flatness. In the example in Fig. 3-3, the plane II is the best fit plane for the points calculated using the LLSE. The planes I and III are two parallel planes (parallel to the plane II) enclosing all the points. According to the definition, the distance between the planes I and III is a minimum. Equivalently, the distance is the sum of the maximum distances between the points on both sides of the fitted plane (plane I) and the plane itself.
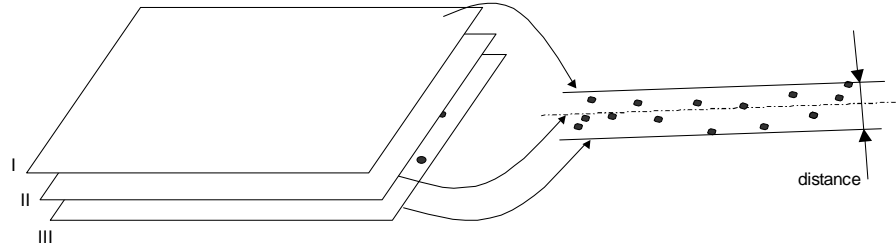
Fig. 3-3 Flatness

Assuming that a point is represented as (x_i, y_i, z_i) and the plane is represented as

($\widehat{x}_0, \widehat{y}_0, \widehat{z}_0$) and ($\widehat{a}, \widehat{b}, \widehat{c}$), the deviation from this point to the plane is:

$$d'_i = \widehat{a}(x_i - \widehat{x}_0) + \widehat{b}(y_i - \widehat{y}_0) + \widehat{c}(z_i - \widehat{z}_0) \qquad\qquad (3\text{-}13)$$

For a set of n points, the distance between two parallel planes is:

$$Dev = \max(d'_i) - \min(d'_i); \qquad\qquad i = 1:n \qquad\qquad (3\text{-}14)$$

The deviation (*Dev*) has to be smaller than the tolerance specification for the flatness.

3.1.3 Parallelism

The parallelism refers to the minimum distance between two parallel planes (or lines) that enclose the fitted surface (or line) of a set of measurement points such that the two planes (or lines) are parallel to a reference plane (or line). The parallelism property for planes can be illustrated by the following Figure (3-4).

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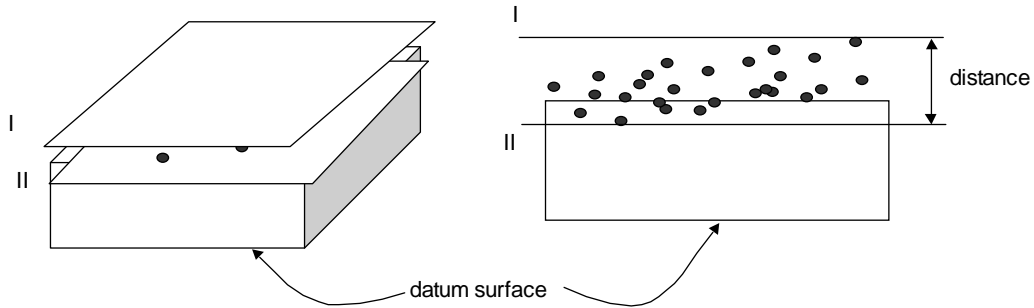
Fig. 3-4 Parallelism

Assuming that a point on the fitted surface is represented as (x_i, y_i, z_i) and the datum plane is represented by a point belonging to it, (x_0, y_0, z_0) and the normal vector, (a, b, c), the distance from this point to the plane is:

$$d_i = a(x_i - x_0) + b(y_i - y_0) + c(z_i - z_0)$$ (3-15)

For a set of *n* points, the distance between the two parallel planes is:

$$Dist = \max(d_i) - \min(d_i); \qquad i = 1:n$$ (3-16)

The distance (*Dist*) has to be smaller than the parallelism specification.

3.1.4   Angularity

Angularity refers to the minimum distance between two parallel planes bounding a feature such that the two planes form a fix angle with respect to a reference plane.
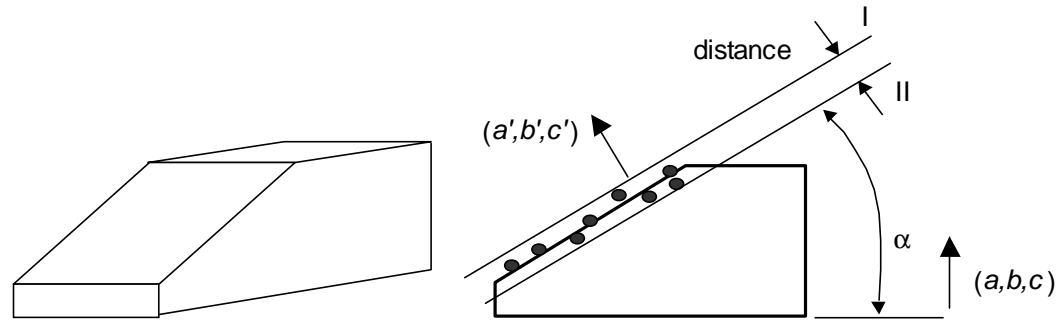
Fig. 3-5 Angularity

The angularity can be calculated given the datum plane (a, b, c, x_0, y_0, z_0), the angle (α), and the surface points (x_i, y_i, z_i). The procedure is the following: (1) rotating the direction vector (a, b, c) with an angle α; this forms a new direction vector (a', b', c'), (2) finding a new fitting surface $(\hat{a}, \hat{b}, \hat{c}, \hat{x}_0, \hat{y}_0, \hat{z}_0)$ for the points (x_i, y_i, z_i), and (3) calculating the minimum distance between two parallel planes (with a direction vector of (a', b', c')) that bound the fitted surface.

3.1.5 Perpendicularity

The perpendicularity tolerance describes how close to perpendicular is one feature relative to another feature [Tolerance, 2004]. It can be applied to a plane or an axis relative to a reference feature. For two theoretically perpendicular planes, the perpendicularity means the allowable distance between two parallel planes that are perpendicular to the datum surface.

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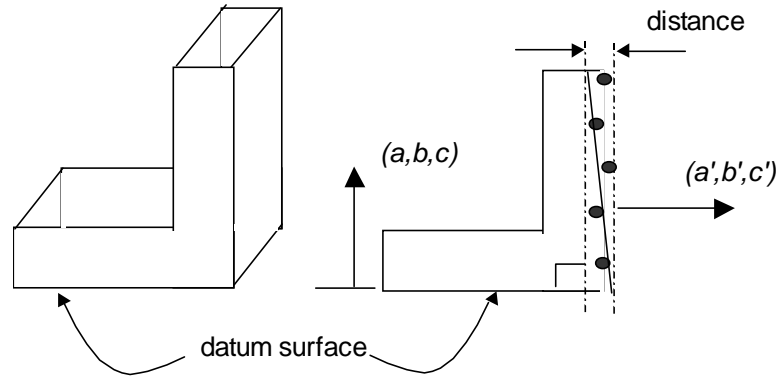
Fig. 3-6 Perpendicularity

The perpendicularity can be calculated given the reference surface (a, b, c, x_0, y_0, z_0) and the surface points (x_i, y_i, z_i), i $=1,\ldots$, n. The procedure is the following: (1) rotating the direction vector (*a, b, c*) with a right angle which forms a new direction vector (*a', b', c'*), (2) fitting a new surface $(\hat{a}, \hat{b}, \hat{c}, \hat{x}_0, \hat{y}_0, \hat{z}_0)$ using the points (x_i, y_i, z_i), i $=1,\ldots$, n, (3) calculating the minimum distance between two parallel planes (with a direction vector of (*a', b', c'*)) that enclose the fitted surface.

3.1.6 Roundness (Circularity)

The roundness or circularity tolerance defines a tolerance zone bounded by two concentric circles so that all the surface elements should lie within this zone. It is generally used to define cylinder shaped surfaces. For instance, the roundness of a cylinder is illustrated in Fig. 3.7 (a). The tolerance zone is depicted as in Fig. 3.7 (b).

The roundness for a cylinder shaped surface is calculated with the following procedure: (1) calculate the best fit cylinder for a given set of points on a cylinder (x_i, y_i, z_i), i $=1,\ldots$, n, and get the parameters of the axis (*a, b, c*), the locating point

(x_0, y_0, z_0), and the diameter R using the LLSE, and (2) calculate the maximum and minimum distances of all surface points to the fitted cylinder axis. The difference between the maximum and minimum distances is estimated as the roundness.

(a)   (b)
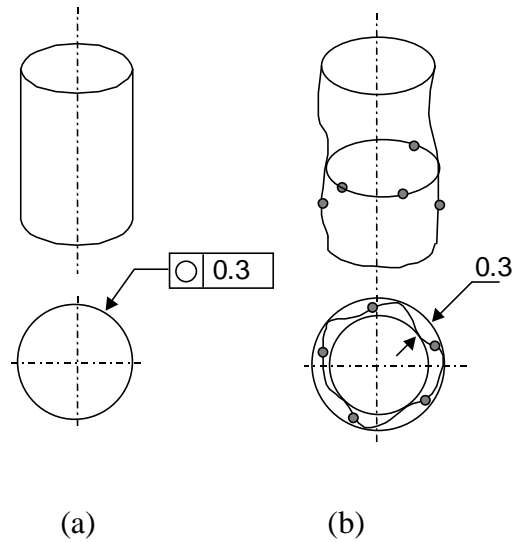
Fig. 3-7 Roundness

3.2   MONTE CARLO SIMULATION FOR TOLERANCE

In the real production, the machining process is usually carried out for a number of products and each product has its own geometric dimensions. In this study, it is convenient to call the set of all of the products as '*population*' and a subset of the population as a '*sample.*' Obviously, the linear least square estimation (LLSE) method in the previous sections calculates the dimensional deviation for a single product. Using the Monte Carlo simulation, it is possible to generate a number of products as a *sample*. For this sample, the mean deviation and the standard deviation of the parameters calculated using the LLSE represent a good estimation of the population characteristics.

To better describe the above calculation, it is reasonable to assume that

(1) the size of the sample is N

(2) the kth product or machined piece has the dimension of $X_k, k = 1:N$

(3) each workpiece has different features for measurements and the tolerances of these features (or their relationships) are represented as $Y_{k,j}$, $j=1$:M, where M is the number of tolerances or measurements.

More specifically, the matrix X_k represents the coordinates of the points of the kth product or the machined piece,

$$X_k = \begin{bmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ ... & ... & ... & ... \\ x_n & y_n & z_n & 1 \end{bmatrix}$$

These points are then used to calculate the parameters of the features on the surface. The features are represented as in the Table 4-1. The tolerances of these features can be calculated from the parameters. For example, the flatness is the minimum distance between two parallel planes (parallel to the best fit plane) that bound all points. By searching all the points, it is easy to calculate the minimum distance. This minimum distance is represented as $Y_{k,j}$.

Based on the above assumption, the dimensional deviation of the jth measurement for a single product and for a number of products can be represented as:

$$Y_{k,j} = function(X_k)$$

$$\overline{Y}_j = \frac{\sum_{k=1}^{N} Y_{k,j}}{N} \qquad\qquad (3\text{-}17)$$

$$\sigma_j = \sqrt{\frac{\sum_{k=1}^{N}\left(Y_{k,j} - \overline{Y}_j\right)}{N-1}}$$

In Eqn. (4-17), the *function* represents the expression found by the Linear Least Square Estimation (LLSE). This method is to calculate the equations for the position, flatness, parallelism, angularity, perpendicularity and roundness. *Y* represents the position, flatness, parallelism, angularity, perpendicularity and roundness (e.g., Δx or *Dist* in Eqn. (4-11), Eqn. (4-13) and Eqn. (4-14)). $\overline{Y}_j$ and σ_j represents the mean and the standard deviation of a certain feature characteristic (e.g., position, flatness, and so on) for the sample.

CHAPTER 4

A NUMERICAL CASE STUDY

The proposed computer aided tolerancing methods can be demonstrated using a numerical case study for the machining process. The development of a case study can be carried out in four steps: (1) constructing a workpiece, (2) setting up the machining process, (3) simulating the machining operation, and (4) calculating the tolerance after machining.

4.1   CONSTRUCTING A WORKPIECE

As shown in Fig. 4-1, a block-shaped workpiece is developed for simulation calculation. The associated dimensions are listed in Table 4-1.
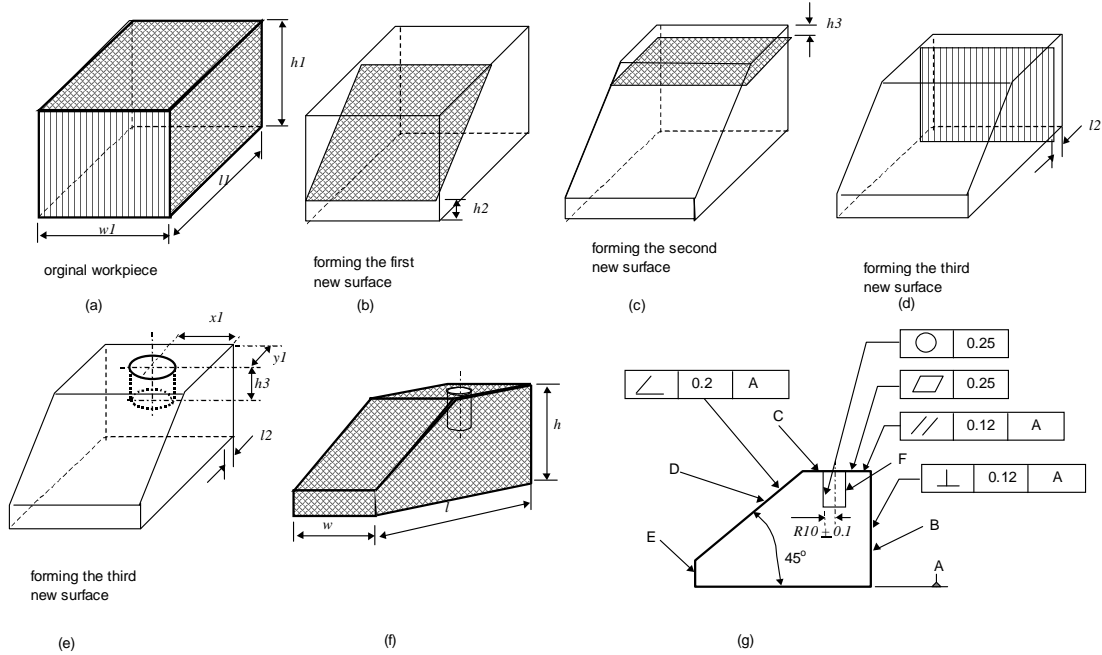
Fig. 4-1 A simulated workpiece and its specification (mm)

Table 4-1 Dimension and Tolerance Specification (Unit: mm), (NS: not-specified)

Variable	w1	l1	h1	h2	h3	L2	w	l	h	x1	y1	h3	R
Nominal value	100	140	110	30	6	6	100	134	94	50	29	30	10
Tolerance	±1	±1	±1	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS

The input workpiece is represented by the surface points, the associated coordinates, and the tolerance. More specifically, assuming that a point on the workpiece has ideal coordinates as $(x_{i,0}, y_{i,0}, z_{i,0})$, the actual point is generated as $(x_{i,0} + \delta x, y_{i,0} + \delta y, z_{i,0} + \delta z)$ and $\delta x, \delta y, \delta z \sim N(0, Tol/Coeff)$, $Tol = 1$ mm, $Coeff = 3$. Here N represents a normal distribution with $\mu = 0$ (the mean) and $\sigma = Tol/Coeff = 0.33$

(the standard deviation), where *Tol* is the maximum allowed tolerance specification and *Coeff* is a control coefficient. The selection of *Coeff* = 3 has been explained in the section 2.2 .This coefficient insures a coverage of 99.73% of the population in the normal distribution.

These generated points are integrated into a matrix, $\bar{X}_0$, to represent the workpiece before machining. Six points from the workpiece are chosen as the locating points and they form a Homogeneous Transformation Matrix for the workpiece, T_w.

4.2 SETTING UP THE MACHINING PROCESS

The workpiece is to be machined three times to generate four new surfaces. The first machining operation mills the surface D, which forms an angle of 45° relative to the bottom surface (Fig. 4-1.b). The second machining operation mills the top surface C, which must be parallel to the bottom surface A (Fig. 4-1.c). The third machining operation mills the side surface B, which must be perpendicular to the bottom surface A (Fig. 4-1.d). The last machining operation drills a hole on the top surface. The specifications are listed in (Fig. 4-1.e and f).

According to the modeling methods, the ideal machined (i.e., new) surface can also be viewed as the intersection of the tool path and the workpiece. The tool path can be represented as a collection of points in the matrix format, $\vec{P}'$. In this case, there are four machining operations and these new surfaces, i.e. $\vec{P}'_D$, $\vec{P}'_C$, $\vec{P}'_B$, $\vec{P}'_H$.

4.3 SIMULATING THE MACHINING OPERATIONS

As presented in the previous sections, the machining operations are carried out on a machining station with a 3-2-1 fixture and a cutting tool. The geometric relationship can be illustrated in Fig. 4-2.
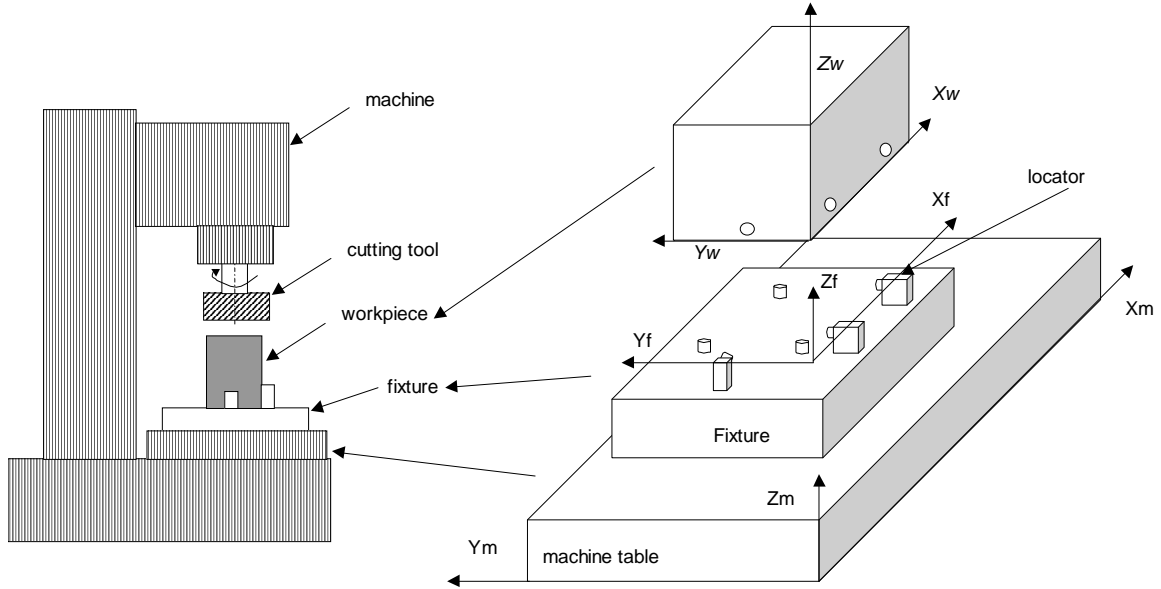

Fig. 4-2 The spatial relationship of the machine, the fixture and the workpiece

The simulation assumes that the variability of the machine, of the fixture, and of the original workpiece are known. This makes it more convenient to construct the homogeneous transformation matrix for the calculation of new surfaces.

The tolerances for the fixture and for the machine are available from the specification or from the measurement. The fixture's tolerance can be specified as the tolerance for the locators' position. For example, if a locator has coordinates $(x_{f,i,0}, y_{f,i,0}, z_{f,i,0})$, then the actual point is generated as $(x_{f,i,0} + \delta x_f, y_{f,i,0} + \delta y_f, z_{f,i,0} + \delta z_f)$ such that $\delta x_f, \delta y_f, \delta z_f \sim N(0, Tol_f / Coeff_f)$, $Tol_f = 0.01$ mm, $Coeff_f = 3$. The selection of $Coeff = 3$ follows the convention of tolerance design (see section 2.2). In a 3-2-1 fixturing scheme, the locators' coordinates form a

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fixture coordinate system (FCS). The variation of the FCS is then represented as a Homogeneous Transformation Matrix (HTM), T_f, following procedures in Eqn. (2-1) to Eqn. (2-7).

The machine's tolerances are often specified as the linear and angular accuracy and precision, where the accuracy refers to the mean shift and the precision refers to the variability. As an example, the specification and the calculation in the X-direction are integrated as follows:

$$
\begin{aligned}
\delta\theta_x &\sim N(Tol_{Angular,Accuracy,x}, 1/Coeff * Tol_{Angular,Precision,x}) \\
\delta x &\sim N(Tol_{Linear,Accuracy,x}, 1/Coeff * Tol_{Linear,Precision,x})
\end{aligned}
\tag{4-1}
$$

The calculated angular and linear deviation are implemented into the HTM, T_m, directly.

4.4   CALCULATING THE TOLERANCE

Based on the HTMs, the workpiece and the tool path representation, the new surface coordinates, $\vec{P}$, can be calculated using the Eqn. (2-12). The new surface coordinates $\vec{P}$ are used to update the workpiece, $\vec{X}_0$, to form a new workpiece coordinate matrix, $\vec{X}_1$. Then Eqn. (3-16) can be applied to the matrix $\vec{X}_1$ to calculate the tolerances such as flatness and angularity.

4.5   SIMULATION RESULTS

A program has been developed in Matlab for the purpose of simulation and calculation. In this study, the simulation calculates N=500 workpieces. The associated tolerances for the N=500 are listed in the following table.

Table 4-2 Simulation results vs. the tolerance specification

Unit (mm)	Simulated Upper Limit	Simulated Lower Limit	Specification in Fig. 4.1
angularity	0.821	0.515	0.2
flatness	0.819	0.542	0.25
parallelism	0.840	0.546	0.12
perpendicularity	0.858	0.560	0.12
R	10.000+0.304	10.000-0.330	0.1
Circularity	0.003	0.001	0.25

From the Table 4-2, it can be observed that most of the simulated tolerances, except the circularity, exceed the specifications. In other words, the tolerance specification cannot be achieved. This provides information for engineers to revise the specification or improve the machining process to reduce the variation.

CHAPTER 5

SUMMARY

In this thesis we have presented model for computing tolaerances for the machining process using CAD tools and simulation. The literature review has shown that there is a need to develop a computer aided tolerancing (CAT) model for the machining process. This study addressed this need and presented a CAD based CAT model for a number of commonly used tolerance specifications, such as flatness, parallelism,

angularity, and so on. The development of the model and the case study have followed four steps:

Step 1: constructing a workpiece

The workpiece is represented as a set of coordinates of individual points on the workpiece surface. These points and their coordinates can be generated by CAD tools or calculation. These coordinates are organized into 4*N (N= the number of points) homogeneous matrixes.

Step 2: setting up the machining process

The machining process removes the original surface on the workpiece and forms a new surface. The machining process, in essence, is a replacement of the original surface with a new surface. Setting up the machining process is to represent the new surfaces with a set of new coordinates, in the form of 4*M matrixes (M= the number of points on the new surfaces). It is similar to the representation of the workpiece.

Step 3: simulating the machining operation

The machining process is simulated by replacing the original surface using the new surface. Meanwhile, the errors from various sources, such as the fixture, the workpiece locating surface, and the machining tool, are considered. The errors are modeled using the 4*4 homogeneous transformation matrix (HTM), whose elements represents the spatial difference between the ideal and the true positions of a spatial vector. The HTM method is commonly used in the field of CAD to calculate the rotation and translation of a spatial vector. With the equations in Chapter 2, the errors in the aforementioned sources can be converted into the newly generated surfaces.

Step 4: calculating the tolerance after machining

The newly generated surfaces can also be represented using different parameters, e.g., a plane can be represented by a point on the surface and a normal vector. These parameters are calculated using the Linear Least Square Estimation method. These parameters are used to calculate the tolerance ranges for different surfaces. These calculated tolerances can be used for engineering improvement.

The above development procedures have been illustrated in this thesis using a simulation case study with the milling and the drilling operations. In the simulation study, a number of tolerances, including the angularity, the flatness, the parallelism, the perpendicularity, the circularity and the dimension, have been calculated. The simulation results can be used to evaluate the engineering design for the tolerance specifications.

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